**Introduction & Background**

• Traditional PDE methods struggle with high-dimensional option pricing problems.

• Neural networks offer mesh-free, flexible alternatives for complex financial models.

• Focus: Three deep learning approaches for option pricing and density estimation.

**Method 1: Deep BSDE**

• **Black-Scholes**: Forward simulation combined with backward optimization to fit Delta, resulting in the initial option price function u(S) at t=0.

• **SABR**: Neural networks fit partial derivatives du/dF and du/dα along forward simulated paths, optimizing the backward stochastic differential equation (BSDE) residual to determine the initial price.

• **Key Advantage**: Learns backward from known terminal payoffs.

**Method 2: Deep Galerkin Method (DGM)**

• **Core Concept**: A mesh-free method directly approximating PDE solutions at randomly sampled points.

• **Black-Scholes**: Utilizes a 5-dimensional neural network u(t,S,r,σ,K), eliminating the need for grid discretization.

• **SABR**: Implements a universal 7-dimensional pricing network u(t,F,α,K,β,ν,ρ) covering broad parameter ranges.

• **Key Advantage**: Efficiently handles high-dimensional spaces, avoiding the curse of dimensionality.

**Method 3: Forward Equation (MDN)**

• **Purpose**: Solves the Fokker-Planck PDE to model the evolution of probability densities.

• **Implementation**: Employs time-sliced Mixture Density Networks (MDNs), each corresponding to a discrete time slice t\_i.

• **Process**: Takes asset price SS as input, outputs a Gaussian mixture model to approximate the empirical density p(t\_i,S).

• **Training**: Uses Monte Carlo-generated reference densities and minimizes Mean Squared Error (MSE).

**Results & Conclusion**

• All three methods effectively approximate solutions to complex option pricing problems.

• Each method is suited to distinct scenarios:

* **Deep BSDE**: backward optimization from known terminal payoffs.
* **DGM**: direct PDE solving in high-dimensional settings.
* **MDN**: precise density estimation through probabilistic modeling.

• Deep learning approaches provide powerful, flexible alternatives to traditional numerical methods in financial modeling.